# AB Calculus Review Sheet #1

**Definition of the derivative of $f(x)$ at any $x$ in the domain of $f$.**

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \text{or} \quad f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}.$$

**Definition of derivative of $f(x)$ at $x = a$**

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \quad \text{or} \quad f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$

**Formal Definition of the limit**

Let $f$ be a function defined on an open interval containing $a$ (except possible at $a$) and let $L$ be a real

number. The statement

$$\lim_{x \to c} f(x) = L$$

means that for each $\varepsilon > 0$ there exists a $\delta > 0$ such that if $0 < |x - c| < \delta$ then $|f(x) - L| < \varepsilon$.

**Existence of a limit**

If $\lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x)$ then $\lim_{x \to c} f(x)$ exists.

**Definition of continuity**

*Continuity at a Point:* A function $f$ is continuous at $c$ if the following three conditions are met.

(1) $f(c)$ is defined. (2) $\lim_{x \to c} f(x)$ exits. (3) $\lim_{x \to c} f(x) = f(c)$.

*Continuity over an interval:* If $f$ is differentiable at $x = c$, then $f$ is continuous at $x = c$. Thus $f$ is continuous wherever $f'$ exists.

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Multiple Choice Questions

1. What is $\lim_{h \to 0} \dfrac{\cos\left(\frac{\pi}{2} + h\right) - \cos\left(\frac{\pi}{2}\right)}{h}$? (A) $-\infty$ (B) $-1$ (C) 0 (D) 1 (E) $+\infty$

2. What is $\lim_{x \to \infty} \dfrac{x^2 - 4}{2 + x - 4x^2}$? (A) $-1$ (B) $-\frac{1}{4}$ (C) $\frac{1}{2}$ (D) 1 (E) The limits does not exist.

3. What is $\lim_{x \to 3} \dfrac{x^2 - x - 6}{x - 3}$? (A) 1 (B) 2 (C) 5 (D) 0 (E) The limits does not exist.

4. The function $f(x) = \ln(\sin x)$ is defined for all $x$ in which of the following intervals?

(A) $0 < x < \pi$ (B) $0 \le x \le \pi$ (C) $\frac{3\pi}{2} < x < \frac{5\pi}{2}$

(D) $\frac{3\pi}{2} \le x \le \frac{5\pi}{2}$ (E) $\frac{3\pi}{2} < x < 2\pi$

5. Let $f$ be defined as follows, where $a \neq 0$.

$$f(x) = \begin{cases} \dfrac{x^2 - a^2}{x - a}, & \text{for } x \neq a \\ 0, & \text{for } x = a \end{cases}$$

Which of the following are true about $f$?

I. $\lim_{x \to a} f(x)$ exists, II. $f(a)$ exists, III. $f(x)$ is continuous at $x = a$.

(A) None (B) I only (C) II only (D) I and II only (E) I, II, and III

6. $\lim_{x \to \infty} \sin(x) =$ (A) $-1$ (B) 0 (C) 1 (D) does not exist (E) none of these

7. $\lim_{x \to 0} \dfrac{\sin x}{x^2 + 3x}$ is (A) 1 (B) $\dfrac{1}{3}$ (C) 3 (D) $\infty$ (E) $\dfrac{1}{4}$

8. If $f$ is continuous for all $x$, which of the following integrals necessarily have the same value?

I. $\int_a^b f(x)dx$ II. $\int_0^{b-a} f(x+a)dx$ III. $\int_{a+c}^{b+c} f(x+c)dx$

(A) I and II only (B) I and III only (C) II and III only

(D) I, II, and III (E) No two necessarily have the same value.

9. $\lim_{x \to 0^-} \dfrac{|x|}{x} =$ (A) 0 (B) 1 (C) $-1$ (D) 2 (E) nonexistent

10. $\lim_{h \to 0} \dfrac{\ln(e+h) - 1}{h}$ is

(A) 0 (B) $\dfrac{1}{e}$ (C) 1 (D) $e$ (E) nonexistent

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11. $\lim\limits_{x\to 0}\frac{|x|}{x}$ is (A) 1 (B) 0 (C) $\infty$ (D) $-1$ (E) nonexistent

12. Given $f(x)=\begin{cases}\frac{x^2-3x+2}{x-2}, & \text{for } x\neq 2\\ a & \text{for } x=2\end{cases}$. If $f$ is continuous at $x=2$, then $a=$

(A) $-2$ (B) $-1$ (C) 0 (D) 1 (E) 2

13. Which of the following functions is NOT everywhere continuous?

(A) $y=|x|$ (B) $y=\frac{x}{x^2+1}$ (C) $y=\sqrt{x^2+8}$

(D) $y=x^{\frac{2}{3}}$ (E) $y=\frac{4}{(x+1)^2}$

14. $\lim\limits_{x\to\infty} x\sin\frac{1}{x}$ is (A) 0 (B) $\infty$ (C) nonexistent (D) $-1$ (E) 1

15. $\lim\limits_{x\to\pi}\frac{\sin(\pi-x)}{\pi-x}$ is

(A) 1 (B) 0 (C) $\infty$ (D) nonexistent (E) none of these

16. $\lim\limits_{x\to\infty}\frac{2^{-x}}{2^x}$ is (A) $-1$ (B) 1 (C) 0 (D) $\infty$ (E) none of these

17. $\lim\limits_{x\to-\infty}\frac{2^{-x}}{2^x}$ is (A) $-1$ (B) 1 (C) 0 (D) $\infty$ (E) none of these

18. If $f(x)=\begin{cases}\frac{x^2-x}{2x} & \text{for } x\neq 0\\ k & \text{for } x=0\end{cases}$ and if $f$ is continuous at $x=0$,

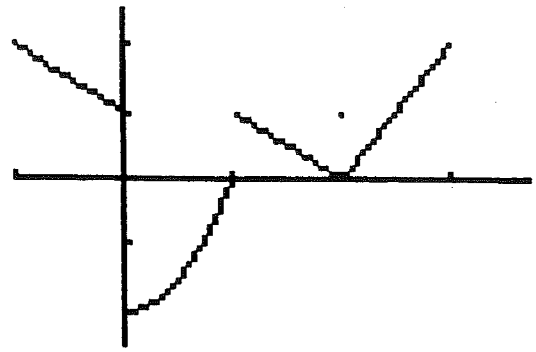
Which of the following statements, I, II, or III, are true?

I. $f(0)$ exists II. $\lim\limits_{x\to 0} f(x)$ exists III. $k=\frac{-1}{2}$

(A) only I (B) only II (C) I and II only (D) I and III only (E) I, II, and III

*Questions 19 through 23 are about the function f shown in the graph and defined below:*

$$f(x) = \begin{cases} 1-x & -1 \le x < 0 \\ 2x^2 - 2 & 0 \le x \le 1 \\ -x+2 & 1 < x < 2 \\ 1 & x = 2 \\ 2x-4 & 2 < x \le 3 \end{cases}$$



19. $\lim_{x \to 2} f(x)$ is (A) 0 (B) 1 (C) 2 (D) does not exist (E) none of these

20. The function $f$ is defined on $[-1,3]$

    (A) except at $x = 0$ (B) except at $x = 1$ (C) except at $x = 2$

    (D) except at $x = 3$ (E) at each $x$ in $[-1,3]$.

21. The function has removable discontinuity at

    (A) $x = 0$ (B) $x = 1$ (C) $x = 2$ (D) $x = 3$ (E) none of these

22. On which of the following intervals is $f$ continuous?

    (A) $-1 \le x \le 0$ (B) $0 < x < 1$ (C) $1 \le x \le 2$

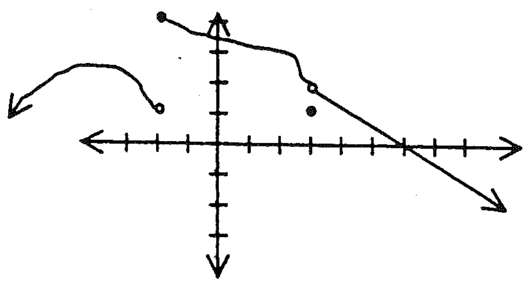
    (D) $2 \le x \le 3$ (E) none of these

23. The function has a jump discontinuity at

    (A) $x = -1$ (B) $x = 1$ (C) $x = 2$ (D) $x = 3$ (E) none of these

Free Response Questions:

1. This is the graph of $f(x)$ to be used with the questions given below.



I. Find the following

a. $\lim_{x\to 0} f(x) =$ ________ b. $\lim_{x\to 3} f(x) =$ ________

c. $\lim_{x\to -2} f(x) =$ ________ d. $\lim_{x\to 6} f(x) =$ ________

e. $\lim_{x\to -2^-} f(x) =$ ________ f. $\lim_{x\to -2^+} f(x) =$ ________

g. $\lim_{x\to 3^+} f(x) =$ ________ h. $\lim_{x\to 3^-} f(x) =$ ________

II. $f(x)$ is discontinuous at ____________________

III. The domain for $f(x)$ is ____________________

IV. The range for $f(x)$ is ____________________

V. Is the inverse of $f(x)$ a function? Explain your answer.

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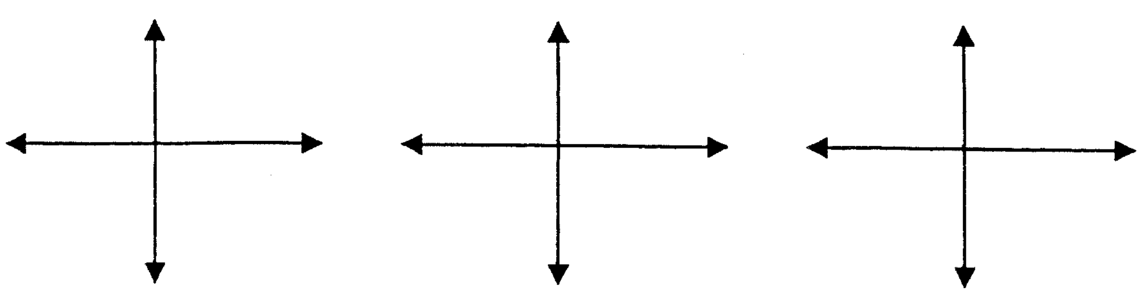
2. (a) What three conditions must be true if $f(x)$ is to be continuous at $x = a$?

(b) Sketch three graphs to illustrate how each condition could fail to be true for $x = a$. Give the condition that is false in each case.



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3. Find the following limits without the use of a calculator.

a. $\lim\limits_{x\to 4}\dfrac{\frac{1}{4x}-\frac{1}{16}}{4-x}=$ ________

b. $\lim\limits_{x\to 0}\dfrac{\sin^2(5x)}{4x^2}=$ ________

c. $\lim\limits_{x\to \infty}\dfrac{\sin(2x)+1}{2x^2}=$ ________

d. $\lim\limits_{x\to \infty}\dfrac{x^2-9}{x+3}=$ ________

e. $\lim\limits_{x\to 5^-}\dfrac{|x-5|}{25-x^2}=$ ________

f. $\lim\limits_{x\to 3} f(x)$ where $f(x)=\begin{cases}2+4x & x\ge 3\\ 5 & x=3\\ 5x-1 & x<3\end{cases}$ is ________

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4. Sketch a graph of the function $g(x)$ that will meet all of the following conditions.

a. $\lim\limits_{x\to 0^-} g(x)=-2$ b. $\lim\limits_{x\to 0^+} g(x)=-2$ c. $f(0)$ does not exist.

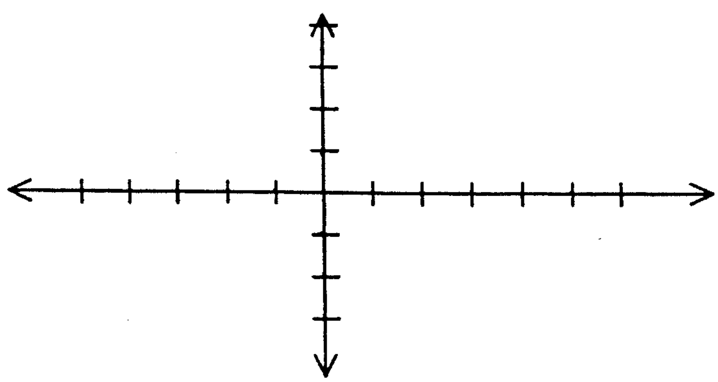
d. $\lim\limits_{x\to 1} g(x)=-1$ e. $g(1)=-1$

f. $\lim\limits_{x\to 2^-} g(x)=0$ g. $\lim\limits_{x\to 2^+} g(x)=1$ h. $g(2)=1$

i. $\lim\limits_{x\to 3^-} g(x)=+\infty$ j. $\lim\limits_{x\to 3^+} g(x)=-\infty$ k. $g(3)$ does not exist.

l. $g(4)=0$ m. $\lim\limits_{x\to 4} g(x)=-1$

n. $\lim\limits_{x\to -\infty} g(x)=0$ o. $\lim\limits_{x\to \infty} g(x)=-\infty$



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5. Let $f$ be a function defined by $f(x)=\begin{cases}2x+1, & \text{for } x\le 2\\ \frac{1}{2}x^2+k, & \text{for } x<2.\end{cases}$

(a) For what value of $k$ will $f$ be continuous at $x=2$? Justify your answer.
(b) Using the value of $k$ found in part (a), determine whether $f$ is differentiable at $x=2$. Use the definition of the derivative to justify your answer.
(c) Let $k=4$. Determine whether $f$ is differentiable at $x=2$. Justify your answer.

# Answers for AB Calculus AP Review

**Review Sheet #1**

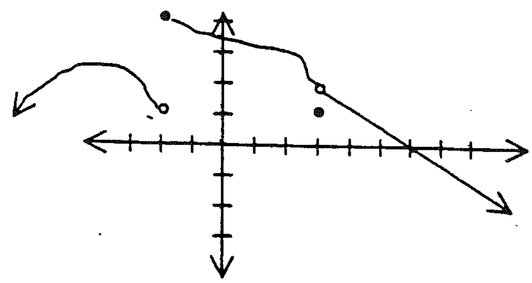
| | | | |
|---|---|---|---|
| 1. B | 7. B | 13. E | 19. A |
| 2. B | 8. A | 14. E | 20. E |
| 3. C | 9. C | 15. A | 21. C |
| 4. A | 10. B | 16. C | 22. B |
| 5. D | 11. E | 17. D | 23. B |
| 6. D | 12. D | 18. E | |

1. See attached
2. See attached
3. See attached
4. See attached
5. 1981 AB5

AB Calculus Review Sheet #1 Answers

Free Response Questions:

1. This is the graph of $f(x)$ to be used with the questions given below.



I. Find the following

a. $\lim_{x\to 0} f(x) =$ 3.5    b. $\lim_{x\to 3} f(x) =$ 2

c. $\lim_{x\to -2} f(x) =$ DNE    d. $\lim_{x\to 6} f(x) =$ 0

e. $\lim_{x\to -2^-} f(x) =$ 1    f. $\lim_{x\to -2^+} f(x) =$ 4

g. $\lim_{x\to 3^+} f(x) =$ 2    h. $\lim_{x\to 3^-} f(x) =$ 2

II. $f(x)$ is discontinuous at $x = -2$, $x = 3$

III. The domain for $f(x)$ is all real numbers

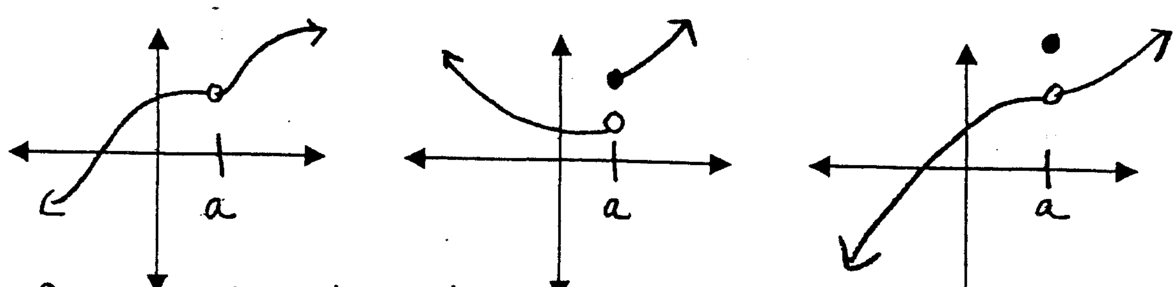
IV. The range for $f(x)$ is $y \le 4$, $y \ne 2$

V. Is the inverse of $f(x)$ a function? Explain your answer. No, y-values are not unique.

2. (a) What three conditions must be true if $f(x)$ is to be continuous at $x = a$?

$f(a)$ exists, $\lim_{x\to a} f(x)$ exists, $\lim_{x\to a} f(x) = f(a)$

(b) Sketch three graphs to illustrate how each condition could fail to be true for $x = a$. Give the condition that is false in each case.




$f(a)$ does not exist    $\lim_{x\to a} f(a)$ does not exist.    $\lim_{x\to a} f(x) \ne f(a)$

3. Find the following limits without the use of a calculator.

a. $\lim_{x\to 4} \frac{\frac{1}{4x}-\frac{1}{16}}{4-x} =$ $\frac{1}{64}$

b. $\lim_{x\to 0} \frac{\sin^2(5x)}{4x^2} =$ $\frac{25}{4}$

c. $\lim_{x\to\infty} \frac{\sin(2x)+1}{2x^2} =$ $0$

d. $\lim_{x\to\infty} \frac{x^2-9}{x+3} =$ $\infty$

e. $\lim_{x\to 5^-} \frac{|x-5|}{25-x^2} =$ $\frac{1}{10}$

f. $\lim_{x\to 3} f(x)$ where $f(x) = \begin{cases} 2+4x & x \ge 3 \\ 5 & x = 3 \\ 5x-1 & x < 3 \end{cases}$ is $14$

4. Sketch a graph of the function $g(x)$ that will meet all of the following conditions.

a. $\lim_{x\to 0^-} g(x) = -2$  b. $\lim_{x\to 0^+} g(x) = -2$  c. $f(0)$ does not exist.

d. $\lim_{x\to 1} g(x) = -1$  e. $g(1) = -1$

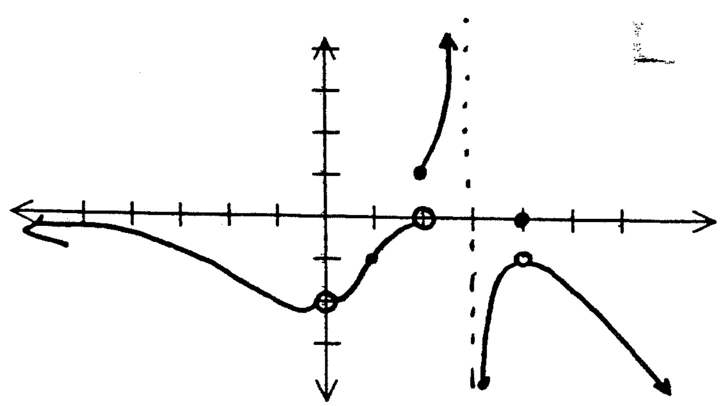
f. $\lim_{x\to 2^-} g(x) = 0$  g. $\lim_{x\to 2^+} g(x) = 1$  h. $g(2) = 1$

i. $\lim_{x\to 3^-} g(x) = +\infty$  j. $\lim_{x\to 3^+} g(x) = -\infty$  k. $g(3)$ does not exist.

l. $g(4) = 0$  m. $\lim_{x\to 4} g(x) = -1$

n. $\lim_{x\to -\infty} g(x) = 0$  o. $\lim_{x\to\infty} g(x) = -\infty$

One possible solution



5. Let $f$ be a function defined by $f(x) = \begin{cases} 2x+1, & \text{for } x \le 2 \\ \frac{1}{2}x^2 + k, & \text{for } x < 2. \end{cases}$

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(a) For what value of $k$ will $f$ be continuous at $x = 2$? Justify your answer.
(b) Using the value of $k$ found in part (a), determine whether $f$ is differentiable at $x = 2$. Use the definition of the derivative to justify your answer.
(c) Let $k = 4$. Determine whether $f$ is differentiable at $x = 2$. Justify your answer.

5. Let f be a function defined by

$$f(x) = \begin{cases} 2x+1, & \text{for } x \le 2, \\ \frac{1}{2}x^2 + k, & \text{for } x > 2. \end{cases}$$

(a) For what values of k will f be continuous at $x = 2$? Justify your answer.

(b) Using the value of k found in part (a), determine whether f is differentiable at $x = 2$. Use the definition of the derivative to justify your answer.

(c) Let $k = 4$. Determine whether f is differentiable at $x = 2$. Justify your answer.

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(a) $\lim_{x \to 2^-} (2x+1) = 5$ $\qquad f(2) = 5$

$\lim_{x \to 2^+} \left(\frac{1}{2}x^2 + k\right) = 2 + k$ $\qquad \therefore \; 2 + k = 5$

$k = 3$

(b) $\lim_{x \to 2^-} \dfrac{f(x) - f(2)}{x-2} = \lim_{x \to 2^-} \dfrac{2x+1-5}{x-2} = 2$

$\lim_{x \to 2^+} \dfrac{f(x) - f(2)}{x-2} = \lim_{x \to 2^+} \dfrac{\frac{1}{2}x^2+3-5}{x-2} = 2$

$\therefore \; f'(2) = 2$ exists.

(c) When $k = 4$,

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \left(\frac{1}{2}x^2 + 4\right) = 6 \neq f(2) = 5.$

Hence f is not continuous at $x = 2$, and thus f is not differentiable at $x = 2$.